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Cutting cheese with wire

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Wire cutting involves fracture, plastic deformation and surface friction effects so that, in principle, it provides a method for determining parameters for all these properties. This paper describes an analysis in terms of the fracture toughness, G_c, the yield stress, σ_Y, and the coefficient of friction, μ. By measuring the cutting force as a function of wire diameter, G_c and $(1 + \mu)\sigma_Y$ can be found. These values are compared with direct measurements in notched bending for G_c, in simple compression for μ and σ_Y, and in sliding tests for μ. A comparison of values obtained for a range of cheeses shows encouraging agreement.

1. Introduction

The mechanical properties of foods are important in determining those eating characteristics usually referred to as texture. Panel test data are also important in defining perceived properties, but they are expensive and time consuming to perform. There is an attraction, therefore, in using conventional mechanical property tests which give material properties which are independent of test methods and are usually easier to perform. There is a profound problem, of course, in relating such properties as yield stress and coefficient of friction to perceptions such as crispness and chewing characteristics and this represents the central challenge of the subject. There is also a preliminary to this stage, however, in that foods are quite difficult to characterize via conventional tests, because of their variability and rather complex viscoelastic nature.

Foods are not the only materials which display such behaviour and it has been widely recognized that methods worked out for surmounting such difficulties in other fields can be usefully applied to foods [1–4]. For example, determining the fracture toughness of foods is quite complex because of a high degree of rate dependence and non-linearity, but judicious use of specimen sizes and loading configurations can give useful results from simple tests [1]. This notion of simplicity is pursued here and applied to the wire cutting of cheese as well as compression and fracture tests. It will be demonstrated that the use of simple analysis and the careful design of the tests yields quantitative agreement in the mechanical parameters measured.

2. Analysis and test methods

The analysis used here is based on the assumption that the textural characteristics of foods can be described using four parameters. These are Young's modulus, E, which defines the elastic response, the yield stress, σ_Y, which defines the stress necessary to induce flow, the coefficient of friction, μ, which determines the surface sliding behaviour, and the fracture toughness, G_c, which describes the energy necessary to create new surfaces. The following testing methods are used.

2.1. Fracture tests

The most convenient method is the single-edge-notched bend (SENB) test as developed for polymers in the ESIS protocol [5]. If the food samples are sufficiently thick, beam samples are prepared such that the width, W, is twice the thickness, B, and a ratio of the span, S, to depth equal to four is achieved. A notch of length, a, is made centrally using a razor blade to give $0.45 < a/W < 0.55$ as shown in Fig. 1. Such deep-notch-bend tests give a high constraint and thus promote brittle fracture. The specimen is loaded to failure at some prescribed rate and the load–displacement diagram recorded. Fracture initiation is determined via a 5% increase in compliance or at maximum load [5] and G_c, is computed from the energy up to this point via

$$G_c = \frac{U}{BW\phi} \quad (1)$$

where U is the energy to fracture and $\phi(a/W)$ is a calibration factor. The application of this method to

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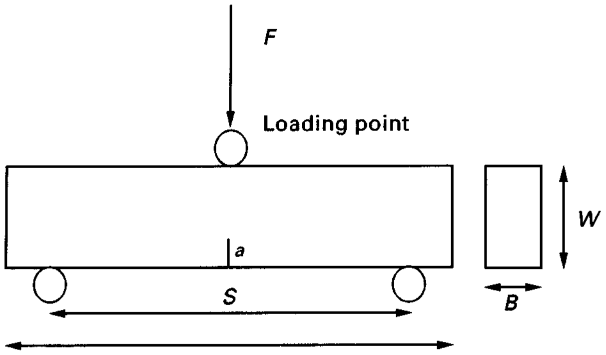

Figure 1 The single-edge-notched bend test.

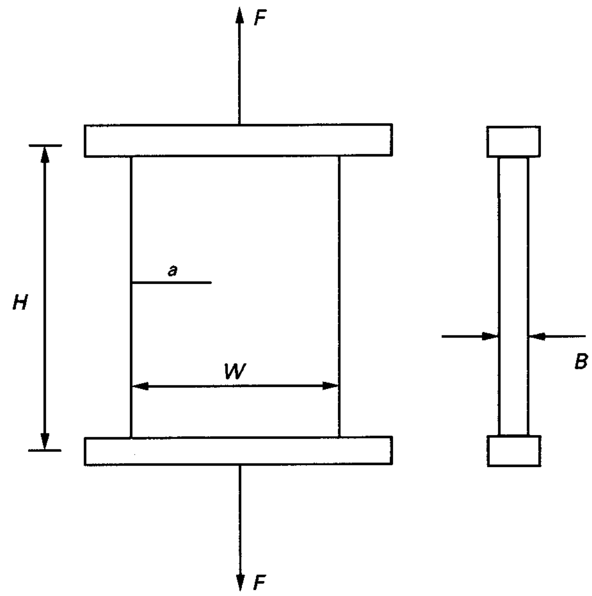

Figure 2 The single-edge-notched tension test.

cheese has been described in detail in [1] and has been shown to be successful in defining G_c.

Some cheeses were only available as slices ($B \approx 3$ mm); so the bend test was not practical. Single-edge-notched tension tests were therefore performed in the geometry shown in Fig. 2. There is no protocol for this configuration; so tests were performed by fracturing specimens for various a/W ratios and determining the fracture stress, σ_f, at initiation. The tests were performed with stiff clamps so that there was effectively no rotation. The fracture toughness can be determined in two ways which provides a useful cross-check on the consistency of the method and non-linear effects. If the fracture stress, σ_f, is measured, then the critical stress intensity factor is given for this rigid clamp case by [6]

$$K_c = 1.12(\pi a)^{1/2}\sigma_f \quad (2)$$

Thus a graph of σ_f versus $[1.12(\pi a)^{1/2}]^{-1}$ should be linear with no intercept and with a slope of K_c. Separate tension tests may be performed to find the tensile modulus, E, and the fracture toughness computed from

$$G_c = \frac{K_c^2}{E} \quad (3)$$

The data may also be analysed in the same manner as the bend tests via energy to failure and ϕ, where ϕ in this case is [6]

$$\phi = \frac{1}{2}\frac{a}{W} + \frac{1}{2.5\pi}\frac{H}{W}\frac{W}{a} \quad (4)$$

In both tests the cheeses did show evidence of non-linearity and rate effects, but the toughness and hence loads are rather low, resulting in minor effects and thus valid results from this linear analysis scheme.

2.2. Compression tests

This a widely used test for characterizing foods. Cylindrical discs with various ratios of height, H, to diameter, d_0, were prepared and then compressed between polished metal plates at fixed rates. If sliding at the surface is assumed, then at any instance at which the stress is σ when the height is h and the diameter d, for a small height change of dh the work input is F dh, where F is the current load as shown in Fig. 3. At a radius r the radial movement, du, is given by

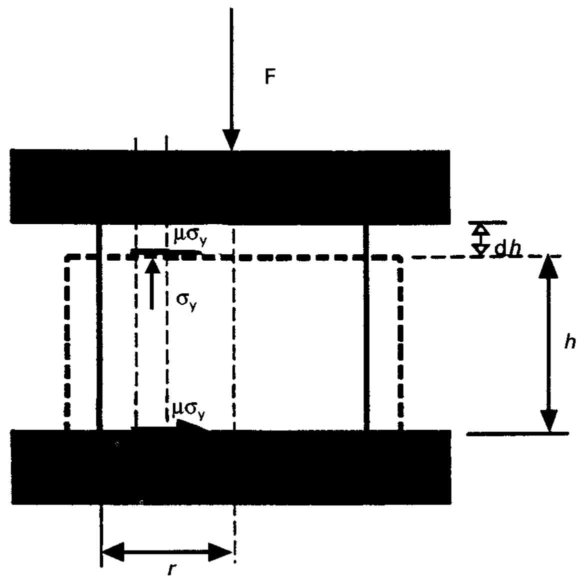

Figure 3 The sample compression test.

$$du = \frac{r}{2h}dh$$

and the total energy is

$$F\,dh = \int_0^{d/2} 2\pi r(\sigma\,dh + 2\mu\sigma\,du)\,dr$$

i.e.,

$$F = \frac{\pi d^2}{4}\sigma + \mu\sigma\frac{\pi d^3}{12h}$$

where μ is the coefficient of friction. Thus the apparent stress σ' is given by

$$\sigma' = \sigma + \frac{\mu}{3}\sigma\frac{d}{h} \quad (5)$$

where $\sigma' = 4F/\pi d^2$. Thus at each height, h, the diameter may be derived from the constant-volume assumption:

$$d^2 h = d_0^2 H \qquad (6)$$

where d_0 and H are the original values and hence

$$\sigma' = \sigma + \frac{\mu}{3}\sigma\left(\frac{H}{h}\right)^{3/2}\frac{d_0}{H} \qquad (7)$$

with

$$\sigma' = \frac{4F}{\pi d_0^2}\frac{h}{H}$$

The test is analysed by taking the curves for various d_0/H values and then plotting σ' as a function of d_0/H at fixed h/H values, which is the usual Cooke–Larke [7, 8] procedure. The intercept gives the true stress, σ, and, from the slope, μ may be found. Thus the data may be used to construct stress–strain curves for each material and also to determine μ as a function of strain. The strain is usually written in the Hencky form, $\varepsilon = -\ln(h/H)$. The curves may then be used to define E, σ and μ as functions of ε in compression.

The analysis used here is the simplest possible and does not involve a yield criterion as in the most common solutions [8, 9]. If the Tresca yield criterion is used, the result is

$$\sigma_y' = 2\sigma_y\left(\frac{h}{\mu d}\right)^2\left[\exp\left(\mu\frac{d}{h}\right) - 1 - \frac{\mu d}{h}\right]$$

which reduces to

$$\sigma_y' = \sigma_y + \frac{\mu}{3}\sigma_y\frac{d}{h}$$

for small $\mu(d/h)$ values, i.e., the same form as Equation 5. The simple form is maintained here and so the result is not confined to yielding or low μ values but is an approximation. It should be noted that the shear stress $\mu\sigma_y$ will be limited to $\sigma_y/2$ for the Tresca criterion and hence $\mu \leqslant \frac{1}{2}$ if this condition pertains.

2.3. Friction tests

A very simple independent method of coefficient-of-friction measurement may be made by the scheme shown in Fig. 4. A cylinder of cheese is loaded with a dead load F_1, and then pulled along the surface as shown where $\mu = F/F_1$. The test may be repeated for various values of F_1 and will give a value for the dynamic coefficient of friction when the block is moving, which may be compared with that given in the compression test discussed above.

2.4. Wire cutting tests

Wire cutting involves fracture, deformation and friction; so it is, in principle, a possible method for measuring these parameters. It has been analysed in considerable detail by Luyten and co-workers [4, 10] who pointed out that the force required for cutting would be proportional to the wire diameter and that there would be a constant component arising from G_c.

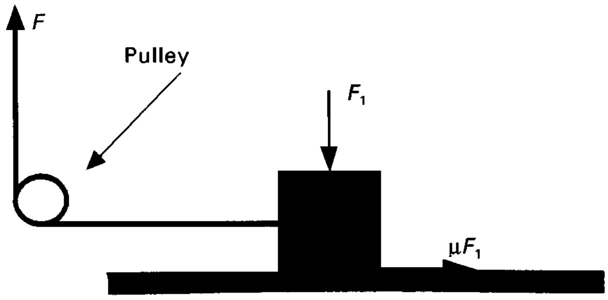

Figure 4 The friction test.

The proportionality factor with diameter would be determined by the deformation energy and the friction. The analysis in [10] is in rather general terms and does not identify μ and σ_y. In fact a simple analysis is possible as shown in Fig. 5a where a wire of diameter, d, is pushed into a wide block of thickness, B. If the cut is too near an edge, then curling can occur which increases the energy dissipation; so here this was avoided.

There are many analyses for elastic frictionless splitting as in cleavage tests (see, for examples, [11]). In that case there is no energy other than G_c; so, if a fracture grows by length dx when the force F moves dx, then

$$F\,dx = G_c B\,dx$$

and

$$G_c = \frac{F}{B} \qquad (8)$$

Where there is yielding and friction, as shown in Fig. 5b, the force F must also sustain σ_y normal to the surface and the frictional force $\mu\sigma_y$. This additional force is

$$F = 2B\int_0^{\pi/2}\frac{d}{2}(\sigma_y\cos\theta + \mu\sigma_y\sin\theta)\,d\theta$$

and the total force is

$$F = BG_c + B(1+\mu)\sigma_y d$$

i.e.,

$$\frac{F}{B} = G_c + (1+\mu)\sigma_y d \qquad (9)$$

Thus, for steady-state cutting, the constant force per unit width, F/B, should be proportional to d with a slope of $(1+\mu)\sigma_y$ and an intercept of G_c.

A more refined version of this analysis is possible in which the plastic strains may be estimated and will be discussed elsewhere [12].

This set of tests allows the four parameters to be measured independently and provides a useful scheme for gaining some insight into their reliability as characterizing parameters. G_c may be found directly from notch tests and then compared with the intercept values derived from the wire cutting tests. The compression tests will provide σ_y and an associated μ value. These may then be compared with those derived from the slopes of the cutting tests. The direct friction tests provide a separate cross-check of μ.

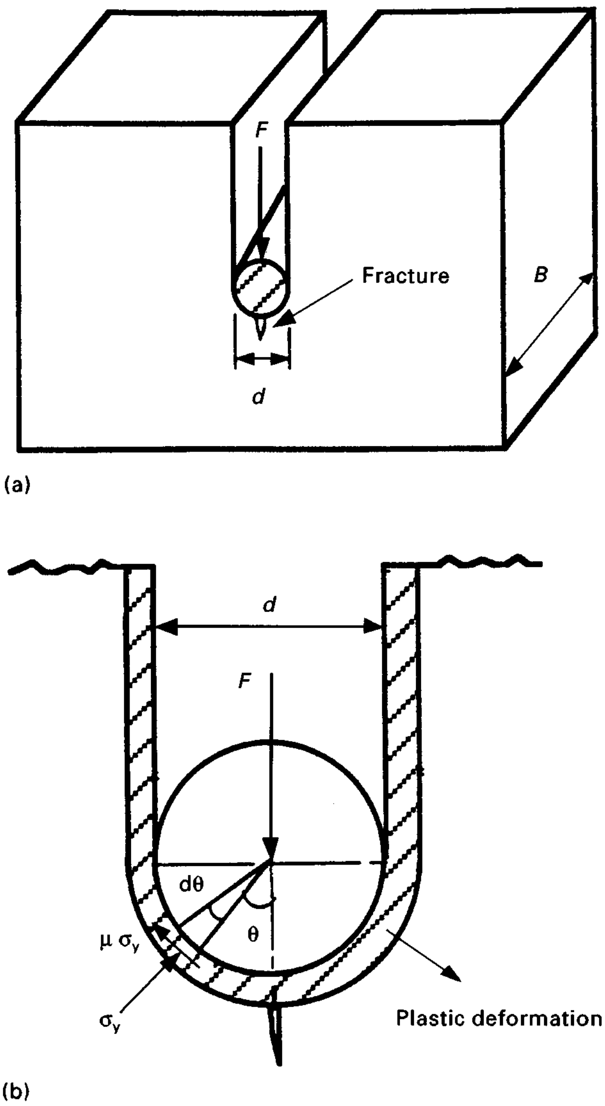

Figure 5 Wire cutting of a block: (a) wire in a block; (b) plastic deformation and frictional forces.

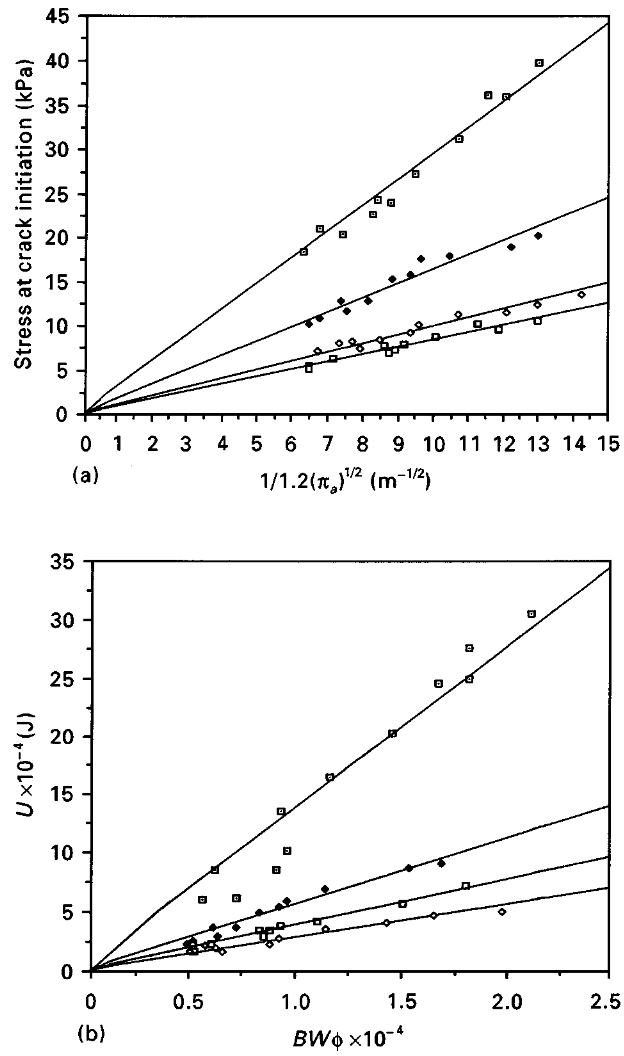

Figure 6 (a) Tension test fracture results using Equation 2. (⊡), mild cheddar, $y = 2.94x$, $R^2 = 0.96$; (◆), American-1, $y = 1.63x$, $R^2 = 0.93$; (♦), Sharp cheddar, $y = 0.98x$, $R^2 = 0.93$); (□), American-2, $y = 0.84x$, $R^2 = 0.95$. (b) Tension test fracture results using Equation 1. (⊡), Mild cheddar, $y = 13.77x$, $R = 0.94$; (◆), American-1, $y = 5.58x$, $R = 0.97$; (□), American-2, $y = 3.82x$, $R = 0.98$; (♦), Sharp cheddar, $y = 2.75x$, $R = 0.94$.

3. Experimental procedure and results

3.1. Materials

Six branded retail cheese products were tested. Two were available as process cheese loaves—a regular and a light variety. The other four were slices: a sharp and a mild cheddar and two American processed cheese varieties. The samples were stored in a refrigerator and then cut and tested at 4 °C. Cheeses are subject to ageing effects [1]; so care was taken to test each material over as short a time span as possible which was usually about 2 days.

3.2. Fracture tests

The two forms of loaf were tested using SENB specimens of dimensions 6.5 mm × 13 mm × 52 mm were tested at 10 mm min^{-1} and the results evaluated using the ESIS protocol [5], as described in [1]. The four slices were tested in tension also at 10 mm min^{-1} and the results in the form of stress versus $[1.12(\pi a)^{1/2}]^{-1}$ are shown in Fig. 6a from which the stress intensity factors can be found and in Fig. 6b as energy versus $BW\phi$. Sample width and length were 14 mm and 56 mm, respectively. The introduced notch lengths were not greater than 7 mm ($W/2$). Fig. 7 shows tensile stress–strain curves from which tensile moduli are found and hence G_c via Equation 3. The values of G_c and the tensile modulus values are given in Table I. For tensile testing, dumb-bell-shaped specimens of 4 mm width and 60 mm gauge length were cut and tested. All tests were performed at 4 °C.

The data show remarkably good linearity in both representations, in spite of evident non-linear characteristics in the stress–strain curves. However, the stress levels of the fracture tests, as indicated in Fig. 7, are quite low and in the linear parts of the curves. The G_c values derived via stress and energy agree reasonably well, i.e., to within ±20%, except for the sharp cheddar sample where the discrepancy is 60%. The values obtained via energy are rather low but for no obvious reason.

TABLE I Fracture toughness, yield stress, slope at cutting test, coefficient of friction and modulus data on six tested cheeses

	G_c (J m^{-2})					μ			E (kPa)	
Sample	Stress	Energy	Cutting	σ_y, compression (kPa)	$(1+\mu)\sigma_y$, cutting (kPa)	Cutting test	Friction test	Compression test	Tension	Compression
Process cheese loaf, regular	—	4.1[a]	4.8	28	49	0.75	0.77–0.85	0.69	—	107
Process cheese loaf, light	—	2.4[a]	3.4	44	82	0.86	0.82–0.93	0.39	—	154
Sharp cheddar	4.5	2.7	5.8	44	91	1.07	0.97–1.04	0.54	220	198
Mild cheddar	11.2	13.8	12.5	84	128	0.52	0.61–0.65	0.54	747	349
Process cheese slice, American-1	6.8	5.6	5.2	62	132	1.13	0.98–1.06	0.57	374	318
Process cheese slice, American-2	4.4	3.8	3.7	32	72	1.25	1.13–1.28	0.53	162	138

[a] Bend test data.

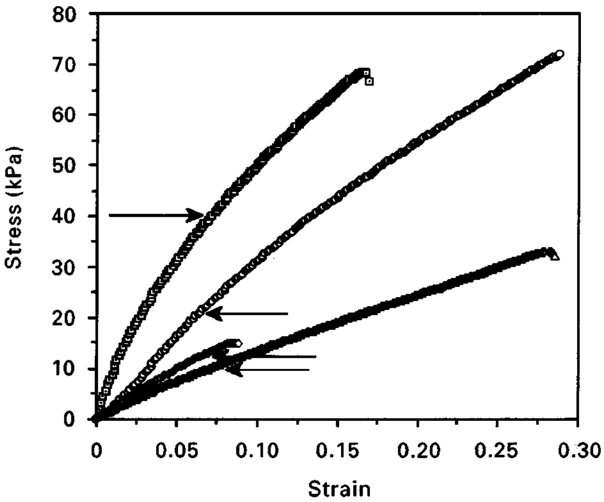

Figure 7 Tension stress–strain curves. (▲), American-2; (⊡), mild cheddar; (○), American 1; (♦), sharp cheddar.
← Maximum stress in fracture tests.

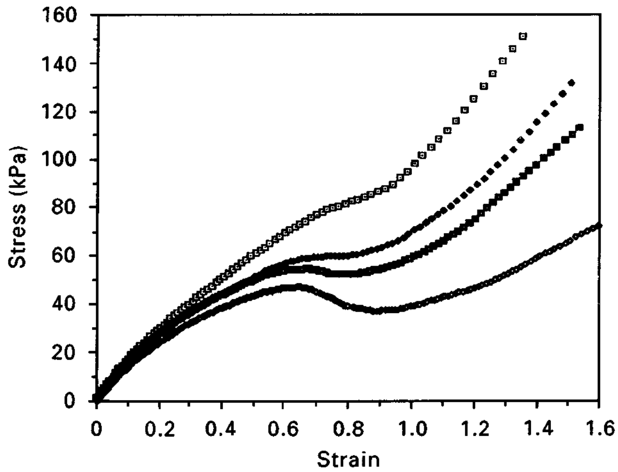

Figure 8 Compression stress–strain curves for American-2 for various sample heights, H. (⊡), 7 mm; (♦) 10 mm; (■), 13 mm; (♦) 20 mm.

3.3. Compression tests

Specimens were prepared for all six materials in the form of cylinders 20 mm in diameter produced using a cylindrical borer. Four heights, 7, 10, 13 and 26 mm, were then prepared by cutting the loaf samples and by stacking several of the slice samples. The tests were performed at 10 mm min^{-1} and at 4 °C. A typical set of stress–strain curves and the extrapolation lines are shown in Figs 8 and 9, respectively and the true stress–strain curve, together with the computed coefficient of friction, is shown in Fig. 10. At low stresses, μ is high because sliding does not occur and μ becomes a minimum at about 0.5 at the maximum stress σ_y. Corrected stress–strain curves for all six materials are given in Fig. 11 and E, σ_y and μ (at σ_y) are given in Table I.

There is about a 10–20% difference between tensile and compression moduli except for mild cheddar where there is a factor of 2 difference. There is no apparent reason for this in the data. The coefficients of friction at the yield point are quite constant and about 0.5 which suggests that there was shear yielding at the interfaces.

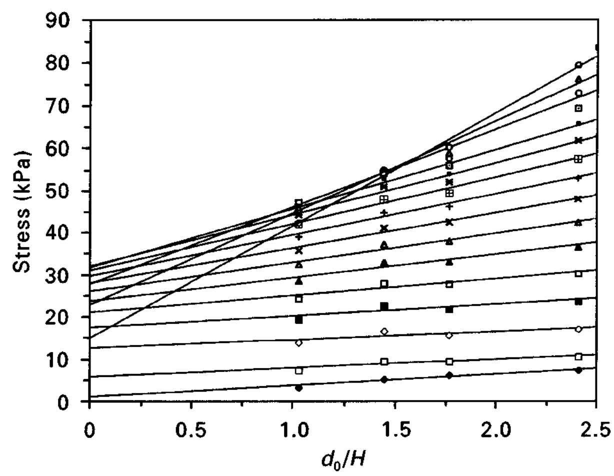

Figure 9 Stress versus (d/H) for American-2 at different strains. (♦), 0.025; (■), 0.05; (♦), 0.1; (■), 0.15; (□), 0.2; (▲), 0.25; (▲), 0.3; (×), 0.35; (+), 0.4; (■), 0.45; (⋇), 0.5; (■), 0.55; (□), 0.6; (●), 0.65; (▲), 0.7; (○), 0.75.

3.4. Friction tests

A typical set of force displacement traces at various loads is shown in Fig. 12 and derived range of values of μ are given in Table I. The values show a variation from about 0.6 to 1.2 with clear distinctions between

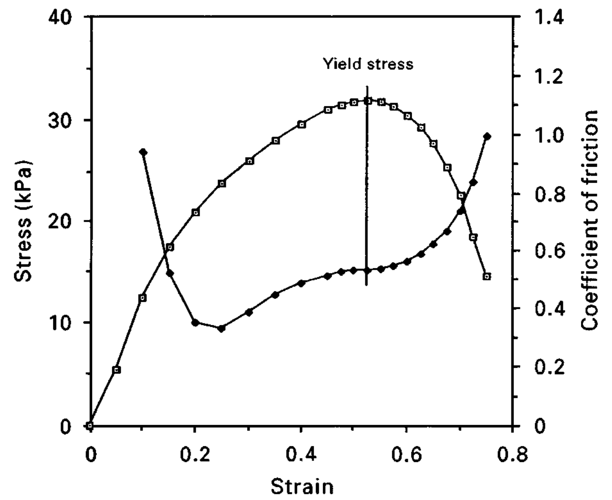

Figure 10 True stress–strain curves (◻) and coefficient of friction (◆) for American-2.

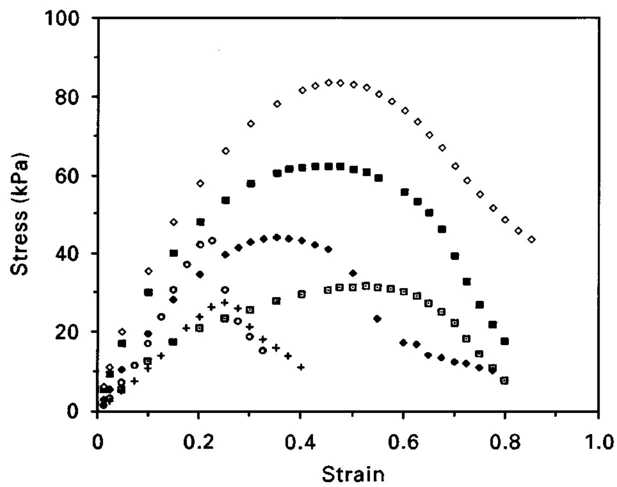

Figure 11 Corrected compressive stress–strain curves. (◻), American-2; (◆), sharp cheddar; (■), American-1; (♦), mild cheddar; (○), light loaf; (+), regular loaf.

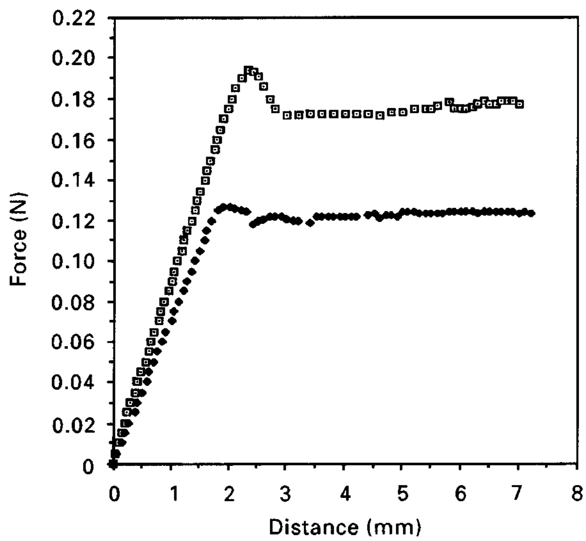

Figure 12 Force–distance curves for the friction test on sharp cheddar. (◻), $\sigma = 0.9$ kPa; (◆), $\sigma = 0.59$ kPa.

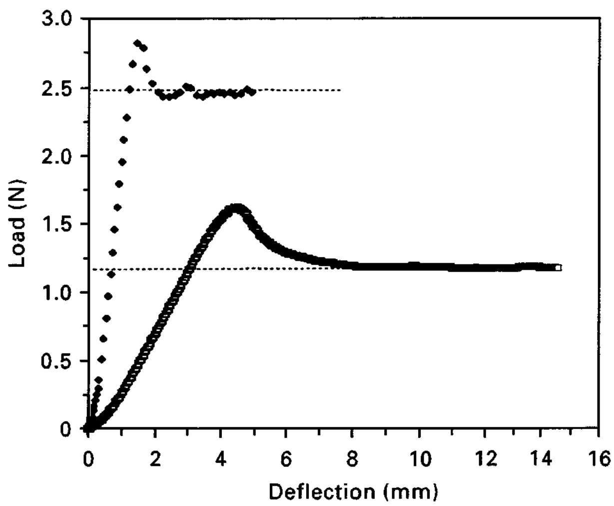

Figure 13 Load–deflection curves for cutting tests. (◻), regular loaf; (◆), American 2.

the materials. The normal stresses were much lower than in the compression tests (i.e. a maximum of 0.9 kPa) and sliding was induced at these values. In the compression tests, sliding occurred under stresses equal to the yield stress.

3.5. Cutting tests

Cutting tests were performed on all six materials using wires of 50, 125, 240, 420, 560, 760 and 980 µm diameters. The width B, of the samples, in Fig. 5 was also varied from 30 to 70 mm for the loaf samples and cuts of about 10 cm were made. Fig. 13 shows typical force–displacement curves and there are clear plateaux which were taken as the steady-state forces.

The averages of five tests per sample was used and the tests were run at 10 mm min^{-1} and 4 °C. Fig. 14 shows a set of data for light loaf and shows good linearity and, although there is some scatter at the larger diameters, the positive intercept is clear and the slope may be defined. For the slice samples, only one width (40 mm) was used and, for wire, diameters of 125, 240, 460 and 780 µm were used; Fig. 15 shows

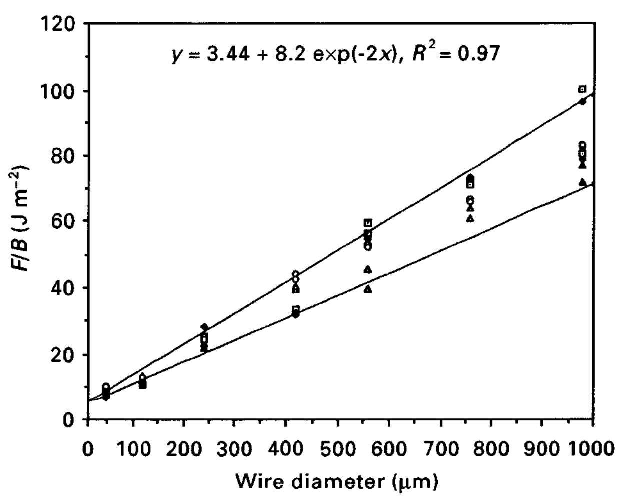

Figure 14 Cutting test data on light process chese loaf for various sample widths, B. (◻), 70 mm; (◆), 60 mm; (▲), 40 mm; (○) 20 mm.

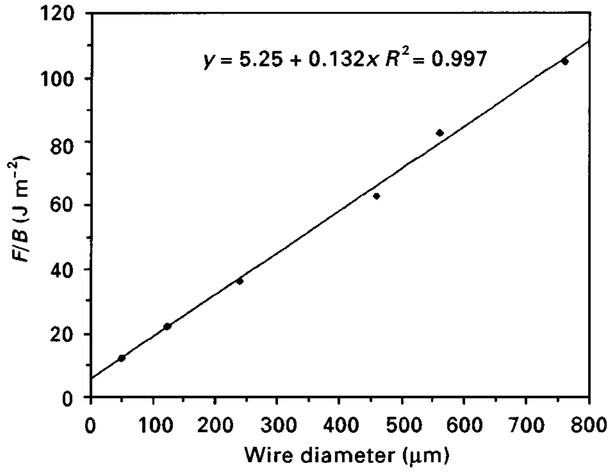

Figure 15 Cutting test data on American-1.

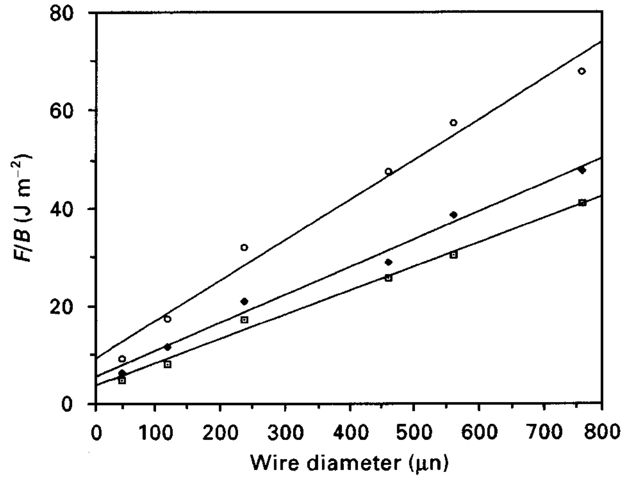

Figure 16 Rate effects in the wire cutting test for regular process cheese loaf. (⊡), 1 mm min^{-1}, $y = 2.85 + 4.95\,\mathrm{e} - 2x$, $R^2 = 0.994$; (◆), 10 mm min^{-1}, $y = 4.59 + 5.70\,\mathrm{e} - 2x$, $R^2 = 0.988$; (O), 100 mm min^{-1}, $y = 8.12 + 8.28\,\mathrm{e} - 2x$, $R^2 = 0.983$).

a typical set of data for mild cheddar. The average slopes $(1 + \mu)\sigma_y$, for all six samples are shown in Table I. The slopes do show some variation with B as indicated in Fig. 14, with higher slopes being associated with wider samples, possibly because edge effects give a lower average stress in narrow samples. For example the light loaf 70 mm samples gave $(1 + \mu)\sigma_y \approx 97$ kPa while for the smaller widths this decreased to 70 kPa with an average value of 82 kPa i.e., a variation of about $\pm 15\%$. The intercept remains sensibly constant.

The G_c values taken from the intercepts are shown in Table I and are generally in quite good agreement with those obtained from the notched tests. For sharp cheddar the agreement is better with the stress-derived value suggesting that the energy value is low. Using the compressive yield stress, together with the slope values, enables μ to be computed and this is shown in Table I. The agreement with the low-stress friction test values is generally very good. This is somewhat surprising since the stresses around the wire would be the yield stress although the average stress would be much lower than in the compression test.

4. Rate effects

Most food products are highly rate dependent and such effects would be expected in fracture and cutting tests. A preliminary study was made using regular process cheese loaf in which the cutting speed was varied between 1 and 100 mm min^{-1} and the results are shown in Fig. 16. There is a clear effect of rate with G_c increasing from 2.8 J m^{-2} at 1 mm min^{-1} to 8.1 J m^{-2} at 100 mm min^{-1}. The slope variation was rather less, being 49.5–82.8 kPa. If the rate dependence is expressed in the form of V^n, then G_c has $n \approx 0.22$ while $(1 + \mu)\sigma_y$ has $n \approx 0.11$. Three-point bend tests performed over the same range of speeds gave $n = 0.44$ for the G_c values. The rather mysterious factors of 2 here are probably coincidence and suggest that there is ample scope for further study. It is gratifying that the cutting test does give the marked rate effects expected.

5. Conclusions

The tests described show that sensible values of the four characterizing parameters for food texture, E, σ_y, G_c and μ, can be obtained. The compression test is needed to obtain σ_y, but the μ value is probably not relevant because it is limited by surface yielding. G_c data obtained via the linear analysis of notched tests, both bending and tension, give sensible values although some cases show large variations which are not understood. The wire cutting test is extremely promising. The G_c values compare very well with those from notch tests, and the slopes, $(1 + \mu)\sigma_y$, appear to correlate well with yielding and friction tests. Indeed the combined parameter may prove useful for defining texture. Rate effects are distinct and easily measured. It seems likely that this test will provide a good mechanical basis for defining texture and avoids the problems of both compression testing and notched fracture tests. It will also, of course, be a useful basis for understanding cutting processes.

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